ALGEBRA SUMMER WORK

Congratulations! You will be in Advanced Algebra when you return to school in September. In order to make the most efficient use of our class time, you are expected to complete this assignment over the summer break.

This packet is due the first day of school. It will count as a quiz grade. The grade will take into account the following:

1. It is complete! There is to be no question left undone. The material in this assignment has been covered in class and you have been provided with clear and thorough explanations. You also have at your disposal wide range of other places for help all over the internet.
2. It is NEAT. You are spending time over the summer to complete this assignment. Turn in something to be proud of. I must be able to easily read your work to assess if it is correct. If I can't read it, it is wrong.
3. The answer you obtained (with supporting work) is correct! Many of the problems required you to show a check for this very reason.

This entire assignment is to be completed without the use of a calculator. You will have the ability to rely on a calculator throughout most of the year starting in September.

The purpose of this assignment is to retain and/or master the skills needed to succeed in Linear Algebra. It will be a demanding course, requiring fluency in all types of numbers. In order to use your calculator effectively, you have to know when to believe if the answer it gives you is correct. That is where much of this assignment comes into play.

When you return in September, you will hand in this assignment and take a diagnostic test. This test will assess your skills and help to determine the progression of our learning for the year as well as highlight any weaknesses that will need to be addressed with further explanation and practice.

Order of Operations

1. Perform any operations inside grouping symbols.
2. Simplify any term with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

EXAMPLE 1: Simplify

$2^2 \div 2 \times (9 - 7) + 8$ **Subtract inside the grouping symbols.**

$2^2 \div 2 \times (2) + 8$ **Simplify exponent.**

$4 \div 2 \times 2 + 8$ **Do multiplication/division in order from left to right.**

$2 \times 2 + 8$ **Do multiplication/division in order from left to right.**

$4 + 8$ **Add.**

12 **The answer is 12.**

DIRECTIONS: No Calculators. Simplify each expression. Evaluate if necessary. Show Work!

1) $2.7 + 3.6 \times 4.5$	2) $3[4(8 - 2) + 5]$	3) $4 + 3(15 - 2^3)$

4) $17 - [(3 + 2) \times 2]$	5) $6 \times (3 + 2) \div 15$	6) $\frac{a+2b}{5}$ for $a = 1$ and $b = 2$
7) $\frac{5m+n}{5}$ for $m = 6$ and $n = 15$	8) $x + 3y^2$ for $x = 3.4$ and $y = 3$	9) $\frac{2(3 + 4)}{7}$
10) $3 + 4[13 - 2(6 - 3)]$	11. 12 | 68 + (-303) | (absolute value bars)	12. $14 + 6 \times 2^3 - 8 \div 2^2$
13. $5 + 4^2 \times 8 - 2^3 \div 2^2$	14. $5(3^2 + 2) - 2(6^2 - 5^2)$	15. $-4^2 - 3^2 - 2(3 - 9)$

Distributive Property

Distributive Property of Addition	$a(b + c) = a \cdot b + a \cdot c$
Distributive Property of Subtraction	$a(b - c) = a \cdot b - a \cdot c$

*** Draw arrows to from the term outside the parentheses to the terms inside to show that the term outside is distributed to each term inside.**

EXAMPLE 1: Simplify by using the Distributive Property

$3(2x + 3)$

$3(2x + 3)$ **Draw arrows.**

$3(2x) + 3(3)$ **Use the Distributive Property.**

$6x + 9$ **Simplify.**

EXAMPLE 1: Simplify by using the Distributive Property

$-(4x + 7)$

$-1(4x + 7)$ **Rewrite using the Multiplication Property of -1**

$-1(4x + 7)$ **Draw arrows.**

$-1(4x) + (-1)(7)$ **Use the Distributive Property**

$-4x - 7$ **Simplify.**

DIRECTIONS: Show Work!... Simplify by using the Distributive Property.

1) $2(5x + 4)$	2) $\frac{1}{4}(12x - 8)$	3) $4(7x - 3)$
4) $-5(4 + 2x)$	5) $6(5 - 3x)$	6) $0.1(30x - 50)$
7) $-\frac{2}{3}(2x - 4)$	8) $(3x + 4)7$	9) $8(x + y)$
10) $-(4x + 3)$	11. $-(-2x + 1)$	12. $-(-6x - 3)$
13. $\frac{2}{5}(5k + 35) - 8$	14. $-4x + 3(2x - 5)$	15. $\frac{3}{4}(8x - 10) - 2(x - 15)$

Operations with Rational Numbers - Addition

✐ To add two numbers with the **same sign**, *add* their absolute values. The sum has the same sign as the addends.
✐To add two numbers with **different signs**, find the *difference* of their absolute values. The sum has the same sign as the addend with the greater absolute value.

EXAMPLES:

SAME SIGNS 6 + 2 *6 + 2* **Find the sum of their absolute values.** 8 **Add.** **8 Since both numbers are positive, the sign of the sum is positive.**	SAME SIGNS -6 + -2 *6 + 2* **Find the sum of their absolute values.** 8 **Add.** **-8 Since both numbers are negative, the sign of the sum is negative.**
DIFFERENT SIGNS -6 + 2 *6 – 2* **Find the difference of their absolute values.** 4 **Subtract.** **-4 Since –6 has the greater absolute value, the sign of the sum is negative.**	DIFFERENT SIGNS -2 + 6 *6 – 2* **Find the difference of their absolute values.** 4 **Subtract.** **4 Since** 6 **has the greater absolute value, the sign of the sum is positive.**

DIRECTIONS: **No** Calculators. Simplify. Evaluate when necessary. Show Work…show the number plugged into the variable first….then evaluate.

1) -3 + (-4)	2) 12 + 5	3) -5 + 8
4) -8 + (-2)	5) -2.3 + (-1.5)	6) 4.5 + 3.1
7) -5.1 + 2.8	8) 13.9 + 7.3	9) -a + (-b) for a = 5 and b = -4
10) -a + b for a = 5 and b = -4	11) a + b for a = 5 and b = -4	12) a + (-b) for a = 5 and b = -4

Operations with Rational Numbers - Subtraction

To subtract a number, first rewrite the problem as an addition sentence by adding the opposite of the second number. Then, follow the rules for addition of numbers.

EXAMPLES:

6 – 2 *6 + - 2* **Rewrite to add the opposite of the number.** **4** **Follow rules for addition.**	-6 – 2 *-6 + -2* **Rewrite to add the opposite of the number.** **-8** **Follow rules for addition.**
-6 – (-2) *-6 + 2* **Rewrite to add the opposite of the number.** **-4** **Follow rules for addition.**	6 – (-2) 6 + 2 **Rewrite to add the opposite of the number.** **8** **Follow rules for addition.**

DIRECTIONS: No Calculators. Simplify. Evaluate when necessary. Show Work!

1) 7 – 12	2) 6 – 9	3) 4 – (-5)
4) 7 – (-3)	5) -3.1 – (-5.4)	6) 8.3 – 5.1
7) -7.8 – 6.6	8) -4.8 – 2.5	9) $a - b$ for $a = -4$ and $b = 3$
10) $-a - b$ for $a = -4$ and $b = 3$	11) $a - (-b)$ for $a = -4$ and $b = 3$	12) $-a - (-b)$ for $a = -4$ and $b = 3$

Operations with Rational Numbers – Multiplication and Division

The product or quotient of two positive numbers is always positive.

The product or quotient The product or quotient of a positive and a negative number is always negative.of two negative numbers is always positive.

EXAMPLES:

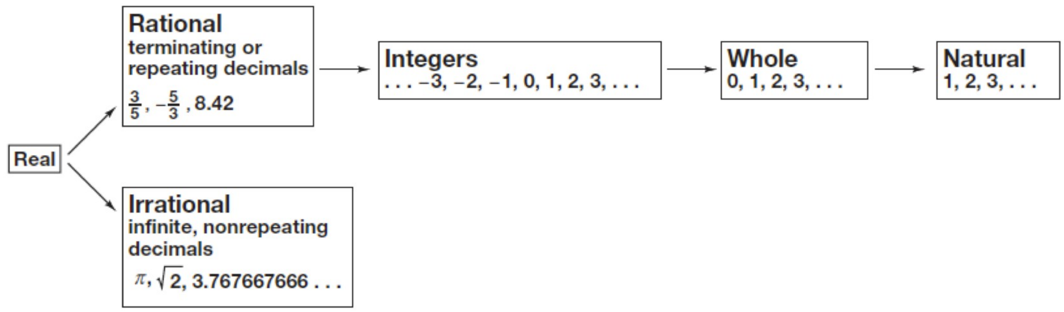
4(2) 8	4(-2) -8
(-4)(-2) 8	(-16) ÷ (-4) 4
16 ÷ (-4) -4	16 ÷ (-4) ÷ (-2) -4 ÷ (-2) 2

DIRECTIONS: No Calculators. Simplify. Evaluate when necessary. Show Work!

1) 4(-2)	2) -6(12)	3) -2(-5)
4) -8(11)	5) $(-7)^2$	6) -10(-5)
7) $-30 \div (-5)$	8) $\frac{-52}{-13}$	9) x^3 for $x = -5$
10) $s^2t \div 10$ for $s = -2$ and $t = 10$	11) $-2m + 4n^2$ for $m = -6$ and $n = -5$	12. $\frac{-2x}{y} - 4y^3$ for x = 6 and y = -2
13. $32 \div (-7 + 5)^3$	14. $-(-4)^3$	15. $(a + b)^2$ for $a = 6$ and $b = -8$

Exploring Real Numbers

The chart below shows the different classifications of real numbers:



EXAMPLES:

Given the numbers $-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, -\pi$ and 32, tell which numbers belong to each set.

Natural:	32	numbers used to count
Whole:	0, 32	natural numbers and zero
Integers:	$0, -9, 32$	whole numbers and their opposites
Rational:	$-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, 32$	integers and terminating and nonrepeating decimals
Irrational:	$-\pi$	infinite, nonrepeating decimals
Real:	$-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, -\pi, 32$	rational and irrational numbers

DIRECTIONS: No Calculators. Name the set(s) of numbers to which each number belongs.

1. -29
2. 7.8
3. 0.384
4. 14.8888.....
5. $0.\overline{57}$
6. $12\frac{4}{5}$
7. $-3\frac{1}{9}$
8. $\sqrt{49}$
9. $\sqrt{10}$
10. $\sqrt{\frac{1}{4}}$

Estimating Square Roots

✐In decimal form, a rational number terminates or repeats.
✐In decimal form, an irrational number continues without repeating.

EXAMPLES:

Complete the following table involving square roots.

Number	Principal Square Root	Negative Square Root	Rational/ Irrational	Perfect Square or $\sqrt{\ }$ Between Which Consecutive Integers
81	9	−9	rational	perfect square
0.25	0.5	−0.5	rational	perfect square
$\frac{4}{9}$	$\frac{2}{3}$	$-\frac{2}{3}$	rational	perfect square
7	2.645 . . .	−2.645 . . .	irrational	between 2 and 3
−17	undefined	undefined	undefined	undefined

DIRECTIONS: No Calculators. For #1, complete the table. For #2–7, simplify each expression, and label it as rational or irrational.

1)

Number	Principal Square Root	Negative Square Root	Rational/ Irrational	Perfect Square or $\sqrt{\ }$ Betwee Which Consecutive Integers
$\frac{1}{64}$				
26				
23				
−36				
$\frac{81}{324}$				

2) $\sqrt{100}$	3) $\sqrt{12}$	4) $\sqrt{-14}$
5) $\sqrt{63}$	6) $-\sqrt{0}$	7) $-\sqrt{\frac{1}{9}}$
Show Your Stuff #1 Between what two consecutive integers is $\sqrt{40}$?	Show Your Stuff #2 Between what two consecutive integers is $\sqrt{139}$?	Show Your Stuff #3 Between what two consecutive integers is $-\sqrt{75}$?

Simplifying Radical Expressions

A radical expression is in **simplest form** when the radicand contains no perfect square factors and the denominator, if applicable, does not contain a radical.

EXAMPLES:

Condition	Not in Simplest Form	How to Simplify	Simplest Form
The Multiplication Property of Square Roots is used to simplify the radical.			
The expression under the radical sign has no perfect square factors other than 1.	$\sqrt{20}$	Rewrite as a product of perfect squares and other factors. $= \sqrt{4 \cdot 5}$ $= \sqrt{4} \cdot \sqrt{5}$	$2\sqrt{5}$
The Division Property of Square Roots is used to simplify the radical.			
The expression under the radical sign is a fraction.	$\sqrt{\frac{16}{25}}$	Separate into two radical expressions. Simplify each separately. $\frac{\sqrt{16}}{\sqrt{25}}$	$\frac{4}{5}$
The denominator contains a radical expression that is not a perfect square	$\frac{3}{\sqrt{2}}$	Rationalize the denominator by multiplying the fraction by a radical expression equal to 1. $= \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$	$\frac{3\sqrt{2}}{2}$

DIRECTIONS: No Calculators. Simplify each radical expression.

1) $\sqrt{22} \cdot \sqrt{8}$	2) $\sqrt{147}$	3) $\sqrt{160}$
4) $\frac{\sqrt{96}}{\sqrt{12}}$	5) $\sqrt{\frac{17}{144}}$	6) $\sqrt{18} \cdot \sqrt{8}$

7) $\sqrt{15} \cdot \sqrt{35}$	8) $\frac{3}{\sqrt{3}}$	Show Your Stuff #1 $\frac{4}{\sqrt{8}}$
Show Your Stuff #2 $\sqrt{3x} \cdot \sqrt{5x}$	Show Your Stuff #3 $\left(2\sqrt{3}\right)^2$	Show Your Stuff #4 $\sqrt{8x^6y^7}$

Two-Step Equations

To isolate the variable,
1. First, eliminate addition or subtraction by using inverse operations.
2. Then, eliminate addition or subtraction by using inverse operations.

EXAMPLE 1:

$3x + 4 = 10$

$\quad -4 \;\; -4$ *Subtraction Property of Equality*

$3x = \quad 6$

$\frac{3x}{3} = \frac{6}{3}$ *Division Property of Equality*

x = 2 *Solution*

CHECK:

$3x + 4 = 10$ *Recopy the original equation*

$3(2) + 4 = 10$ *Substitute the solution for the variable*

$6 + 4 = 10$ *Use Order of Operation to simplify*

$10 = 10$ *Make sure values match*

EXAMPLE 2:

$1 = -\frac{-k}{5} - 3$

$+3 \qquad +3$ *Addition Property of Equality*

$4 = -\frac{-k}{5}$

$4(5) = -\frac{-k}{5}(5)$ *Multiplication Property of Equality*

20 = -(-k)

20 = k

CHECK:

$1 = -\frac{-k}{5} - 3$ *Recopy the original equation*

$1 = -\frac{-20}{5} - 3$ *Substitute the solution for the variable*

5

$1 = -(-4) - 3$ *Use Order of Operation to simplify*

$1 = 4 - 3$

$1 = 1$ *Make sure values match*

DIRECTIONS: No Calculators. Solve each equation. Check your solution.

1) $5a + 2 = 7$	2) $3x – 7 = 35$	2) $67 = -3y + 16$
#1 CHECK	#2 CHECK	#3 CHECK
4) $4s – 13 = 51$	5) $5.8n + 3.7 = 29.8$	6) $11.6 + 3a = -16.9$

#4 CHECK	#5 CHECK	#6 CHECK
7) $\frac{k}{3} - 19 = -26$	8) $\frac{x}{2} + 8 = -3$	9) $-9 = -\frac{x}{12} + 5$
#7 CHECK	#8 CHECK	#9 CHECK
10. $\frac{3}{5}x - \frac{1}{2} = 2\frac{1}{2}$	#10 CHECK	

Multi-Step Equations

To isolate the variable,

1. Distribute if necessary.Combine like terms on one side of the equation, if necessary. [*This should look like a two-step equation now!*]
2. Eliminate addition or subtraction by using inverse operations.
3. Eliminate multiplication or division by using inverse operations.

EXAMPLE 1:

$3c - 8c + 7 = -18$

$-5c + 7 = -18$ *Combine like terms*

$-7 \quad -7$ *Subtraction Property of Equality*

$-5c = -25$

$\frac{-5c}{-5} = \frac{25}{-5}$ *Division Property of Equality*

C = -5

EXAMPLE 2:

$-4d + 2(3 + d) = -14$

$-4d + 2 \cdot 3 + 2 \cdot d = -14$ *Use the Distributive Property*

$-4d + 6 + 2d = -14$

$-2d + 6 = -14$ Combine like terms

$-6 \quad -6$ Subtraction property of equality

$-2d = -20$

$\frac{-2d}{-2} = \frac{-20}{-2}$ *Division Property of Equality*

d = 10 Solution

CHECK:

$3c - 8c + 7 = -18$ *Recopy the original equation*

$3(5) - 8(5) + 7 = -18$ *Substitute solution for the variable*

$15 - 40 + 7 = -18$ *Use Order of Operation to simplify*

$-25 + 7 = -18$

$-18 = -18$ *Make sure values match*

CHECK:

$-4d + 2(3 + d) = -14$ *Recopy the original equation*

$-4(10) + 2(3 + (10)) = -14$ *Substitute solution for the variable*

$-40 + 2(13) = -14$ *Use Order of Operation to simplify*

$-40 + 26 = -14$

$-14 = -14$ *Make sure values match*

Directions: No Calculators. Solve each equation. Check your solution.

1) $2n + 3n + 7 = -41$	2) $2b - 6 + 3b = 14$	3) $3(t - 12) = 27$
#1 CHECK	#2 CHECK	#3 CHECK

4) $2(a-4)+15=13$	5) $4=0.4(3d-5)$	5) $\frac{1}{2}(12n-8)=26$
#4 CHECK	#5 CHECK	#6 CHECK
7. $\frac{2}{3}g+\frac{1}{2}g=14$	8. $\frac{3x+2}{7}=8$	9. $\frac{5x-3}{6}=\frac{2x-1}{3}$
#7 CHECK	#8 CHECK	#9 CHECK

Equations with Variables on Both Sides

To isolate the variable,

1. Distribute if necessary.
2. Combine like terms on *each* side of the equation, if necessary.
3. Get the variable on one side of the equation by using inverse operations (either addition or subtraction). [*The equation should look like a two-step equation now!*]
4. Eliminate addition or subtraction by using inverse operations. Eliminate multiplication or division by using inverse operations.

EXAMPLE: Solve and check.

$3k + 5 = 4(k + 1)$

$3k + 5 = 4k + 4$ *Use the Distributive Property*

$-4k \quad -4k$ *Subtraction Property of Equality**

$-k + 5 = 4$

$-5 \quad -5$ Subtraction property of Equality

$-k = -1$

$k = 1$

CHECK:

$3k + 5 = 4(k + 1)$ *Recopy the original equation*

$3(1) + 5 = 4((1) + 1)$ *Substitute solution for the variable*

$3 + 5 = 4(2)$

$8 = 8$

* *Another option is to subtract 3k from both sides. The resulting equivalent equation would be* $5 = k + 4$. *Both equations will lead to the same solution.*

DIRECTIONS: No Calculators. Solve and graph each inequality. Include a check.

1) $7 - 2n = n - 14$	2) $3d + 8 = 2d - 7$	3) $2(6 - 4d) = 25 - 9d$
#1 CHECK	#2 CHECK	#3 CHECK

4) $2(4 - 2r) = -2(r + 5)$	5) $3.6y = 5.4 + 3.3y$	6) $4(b - 1) = -4 + 4b$
#4 CHECK	#5 CHECK	#6 CHECK
7) $\frac{3}{4}t = \frac{5}{6}t - \frac{2}{3}$	8) $8 - 3(p - 4) = 2p$	9) $5x + 2(1 - x) = 2(2x-1)$
#7 CHECK	#8 CHECK	#9 Check

One-Step Inequalities – Addition and Subtraction

To solve an inequality with addition and subtraction, follow the same procedure for equations to isolatethe variable. There are infinitely many solutions to an inequality. To represent the solution set, the solution to the inequality is graphed on a number line.

For $\le$ and $\ge$,a **closed** circle is placed on the boundary value since the value **is** part of the solution set. For <, >, and $\neq$ an **open** circle is placed on the boundary value since the value **is not** part of the solution set

To represent the remainder of the solution set, shade the segment of the number line that satisfies the values of the solution set.

To check the solution set, selectis shaded in the graph). Substitute to ensure the final inequality statement is true. a value that satisfies the inequality (this will be a number that

EXAMPLE 1:

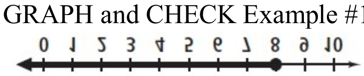
Solve $y - 10 \le -2$

$+10 \quad +10$ *Addition Property of Inequality*

$y \le 8$ *Solution*

** Any value that is less than or equal to 8 is part of the solution set.*

GRAPH and CHECK Example #1



$y - 10 \le -2$ *Recopy the original inequality*

$7 - 10 \le -2$ *Substitute a value from the solution set**

$-3 \le -2$ *Make sure the inequality statement is true*

** Any value that is "shaded in" on the graph could be used to check the solution.*

EXAMPLE 2: Solve

$12 < w + 16$

$-16 \quad -16$ *Subtraction Property of Inequality*

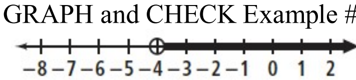
$-4 < w$ *Solution*

$w > -4$

***An equivalent inequality where the variable is on the right can also be a representation of the solution set. Notice that the inequality symbol must flip!*

** Any value that is less than or equal to -4 is part of the solution set.*

GRAPH and CHECK Example #2



$12 < w + 16$ *Recopy the original inequality*

$12 < -3 + 16$ *Substitute a value from the solution set**

$12 < 13$ *Make sure the inequality statement is true*

** Any value that is "shaded in" on the graph could be used to check the solution.*

DIRECTIONS: No Calculators. Solve and graph each inequality. Include a check.

1) $n - 7 \ge 2$	2) $x + 1 \le -3$	3) $d - 13 \le -8$

#1 GRAPH AND CHECK	#2 GRAPH AND CHECK	GRAPH AND CHECK #3
4) $a + 15 > 19$	5) $x - \frac{3}{4} \geq \frac{1}{2}$	$13 < 8 + k - 6$
GRAPH AND CHECK #4	GRAPH AND CHECK #5	GRAPH AND CHECK #6

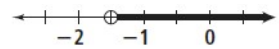
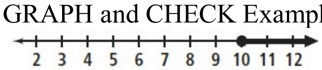
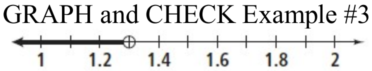
One-Step Inequalities – Multiplication and Division

To solve an inequality with multiplication and division, follow the same procedure for equations to isolate the variable.

There are infinitely many solutions to an inequality.

When you multiply or divide an inequality by a **negative** number, you must ***flip the inequality symbol.***

To represent the solution set, follow the same procedure explained in the previous lesson to graph the solution set on a number line.

EXAMPLE 1: Solve $\frac{b}{2} > -\frac{3}{4}$ $2 * \frac{b}{2} > -\frac{3}{4} * 2$ *Multiplication Property of Inequality* $b > -1.5$ *Solution* ** Any value that is greater than -1.5 is part of the solution set.*	GRAPH and CHECK Example #1  $\frac{b}{2} > -\frac{3}{4}$ *Recopy the original inequality* $\frac{-1}{2} > -\frac{3}{4}$ *Substitute a value from the solution set** $-\frac{1}{2} > -\frac{3}{4}$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*
EXAMPLE 2: Solve $\frac{-100c}{-100} \leq \frac{-1,000}{-100}$ *Division Property of Inequality* $p \geq 10$ *Solution* *** Notice that the inequality symbol must **flip** because you divide both sides of the inequality by **negative** number to isolate the variable!* ** Any value that is greater than or equal to 10 is part of the solution set.*	GRAPH and CHECK Example #2  $-100c \leq -1,000$ *Recopy the original inequality* $-100(10) \leq -1,000$ *Substitute a value from the solution set** $-1,000 \leq -1,000$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used for to check the solution.*
EXAMPLE 3: Solve $\frac{3g}{3} < \frac{3.9}{3}$ *Division Property of Equality* $q < 1.3$ *Solution* ** Any value that is less than 1.3 is part of the solution set.* *Note: There is no need to flip the inequality symbol because you are dividing both sides by positive 3.*	GRAPH and CHECK Example #3  $3g < 3.9$ *Recopy the original inequality* $3(1) < 3.9$ *Substitute a value from the solution set** $3 < 3.9$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*

DIRECTIONS: No Calculators. Solve and graph each inequality. Include a check.

1) $-\frac{n}{7} \geq 6$	2) $2.6v > 6.5$	3) $-\frac{t}{3} < 5$
GRAPH AND CHECK #1	GRAPH AND CHECK #2	GRAPH AND CHECK #3
4) $-7c < 28$	5) $60 \leq 12b$	6) $17 < \frac{p}{2}$
GRAPH AND CHECK #4	GRAPH AND CHECK #5	GRAPH AND CHECK #6

7) $0.9 \leq -1.8v$	8) $0 < \frac{w}{7}$	9) $-\frac{4}{5}r \leq 8$
GRAPH AND CHECK #7	GRAPH AND CHECK #8	GRAPH AND CHECK #9
10) $-\frac{2}{3}x \geq \frac{8}{9}$	11) $\frac{15}{8} \leq \frac{5x}{2}$	12) $-\frac{1}{9}x > \frac{1}{3}$
GRAPH AND CHECK #10	GRAPH AND CHECK #11	GRAPH AND CHECK #12

Multi-Step Inequalities

To solve a multi-step inequality, follow the same procedure for solving equations to isolate the variable. Recall that there are infinitely many solutions to an inequality.

Remember that when you multiply or divide an inequality by a **negative** number, you must ***flip the inequality symbol.***

EXAMPLE: Solve	CHECK:
$\frac{1}{2}(2t+8) \ge 4+6t$ $t+4 \ge 4+6t$ *Distributive Property* $-6t \quad -6t$ *Subtraction Property of Equality* $-5t+4 \ge 4$ $-4 \quad -4$ *Subtraction Property of Equality* $\frac{-5t}{-5} \ge \frac{0}{-5}$ *Division Property of Equality* $t \le 0$ *Solution* *** Notice that the inequality symbol must **flip** because you divide both sides of the inequality by **negative** number to isolate the variable!* ** Any value that is less than or equal to 0 is part of the solution set.*	$\frac{1}{2}(2t+8) \ge 4+6t$ *Recopy the original inequality* $\frac{1}{2}(2(0)+8) \ge 4+6(0)$ *Substitute a value from the solution set** $\frac{1}{2}(0+8) \ge 4+6(0)$ $\frac{1}{2}(8) \ge 4+6(0)$ $4 \ge 4+6(0)$ *Use Order of Operation to simplify* $4 \ge 4+0$ $4 \ge 4$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*

DIRECTIONS: No Calculators. Solve each inequality. Include a check.

1) $2h - 13 < -3$	2) $-4p + 28 > 8$	3) $4(k - 1) > 4$
CHECK #1	CHECK #2	CHECK #3

4) $13t - 8t > -45$	5) $6u - 18 - 4u < 22$	6) $2z + 7 < z + 10$
CHECK #4	CHECK #5	CHECK #6
7) $8m - 8 \geq 12 + 4m$	8) $h + 2(3h + 4) \geq 1$	9) $\frac{1}{2}x - x > -\frac{1}{3}x$
CHECK #7	CHECK #8	CHECK #9

10) $3.4 + 1.6v < 5.9 - 0.9v$	11) $2(3 + 3g) \geq 2g + 14$	12) $3(4g - 6) \geq 6(g + 2)$
CHECK #10	CHECK #11	CHECK #12

Ratios and Proportions

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y, $x{:}y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 **Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.**

$\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.

$\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 **Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.**

$\frac{10}{18} \stackrel{?}{=} \frac{25}{45}$	Write the proportion.
$10(45) \stackrel{?}{=} 18(25)$	Cross products
$450 = 450$	Simplify.

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Practice: Determine if each pair of ratios forms a proportion

1. $\frac{12}{32}, \frac{3}{16}$

2. $\frac{15}{20}, \frac{9}{12}$

3. $\frac{1.5}{2}, \frac{6}{8}$

4. 5 to 9, 25 to 45

5. 0.1 to 0.2, 0.45 to 1.35

6. 100:75, 44:33

Ratios and Proportions

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes.** They are the first and last terms of the proportion. 5 and 10 are called **means.** They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions	For any numbers a, b, c, and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example

$$\frac{x}{5} = \frac{10}{13}$$

$$x \cdot 13 = 5 \cdot 10$$

$$13x = 50$$

$$\frac{13x}{13} = \frac{50}{13}$$

$$x = \frac{50}{13}$$

Example 2:

$$\frac{x+1}{4} = \frac{3}{4}$$

$$4(x+1) = 3 \cdot 4$$

$$4x + 4 = 12$$
$$-4 \quad -4$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Practice: Solve and check each proportion

Solve	Check
1. $\frac{x}{21} = \frac{3}{63}$	
2. $\frac{-3}{x} = \frac{2}{8}$	

3. $\frac{0.1}{2} = \frac{0.5}{x}$	
4. $\frac{9}{y+1} = \frac{18}{54}$	
5. $\frac{a-8}{12} = \frac{15}{3}$	
6. $\frac{3+x}{4} = \frac{-x}{8}$	

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.
original: 48
new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$\frac{12}{48} = \frac{r}{100}$	Percent proportion
$12(100) = 48(r)$	Cross products
$1200 = 48r$	Simplify.
$\frac{1200}{48} = \frac{48r}{48}$	Divide each side by 48.
$25 = r$	Simplify.

The percent of increase is 25%.

Example 2

Find the percent of decrease.
original: 30
new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$\frac{8}{30} = \frac{r}{100}$	Percent proportion
$8(100) = 30(r)$	Cross products
$800 = 30r$	Simplify.
$\frac{800}{30} = \frac{30r}{30}$	Divide each side by 30.
$26\frac{2}{3} = r$	Simplify.

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Practice: State whether each percent of change is a percent of increase or percent of decrease. Then find each percent of change.

1. Original: 50
   New: 80

2. Original: 14.5
   New 10

3. Original: 27.5
   New: 25

4. Original: 250
   New: 500

Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example **A coat is on sale for 25% off the original price. If the original price of the coat is $75, what is the discounted price?**

The discount is 25% of the original price.

25% of $75 = 0.25 × 75   25% = 0.25
= 18.75   Use a calculator.

Subtract $18.75 from the original price.
$75 − $18.75 = $56.25

The discounted price of the coat is $56.25.

Practice: Find the final price of each item. When a discount and sales tax are listed, compute the discount price before computing the tax.

1. Two concert tickets: $28
   Student discount: 28%

2. Airline ticket: $248
   Frequent flyer discount: 33%

3. CD player: $142
   Sales tax: 5.5%

4. Celebrity calendar: $10.95
   Sales tax: 7.5%

5. Camera: $110.95; Discount: 20%
   Sales tax: 5%

6. Ipod: $89; Discount: 17%
   Tax: 5%

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 **Solve $2x - 4y = 8$ for y.**

$$2x - 4y = 8$$
$$2x - 4y - 2x = 8 - 2x$$
$$-4y = 8 - 2x$$
$$\frac{-4y}{-4} = \frac{8 - 2x}{-4}$$
$$y = \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 **Solve $3m - n = km - 8$ for m.**

$$3m - n = km - 8$$
$$3m - n - km = km - 8 - km$$
$$3m - n - km = -8$$
$$3m - n - km + n = -8 + n$$
$$3m - km = -8 + n$$
$$m(3 - k) = -8 + n$$
$$\frac{m(3 - k)}{3 - k} = \frac{-8 + n}{3 - k}$$
$$m = \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Practice: Solve each equation or formula for the variable specified

1. $15x + 1 = y$ for x
2. $x(4 - k) = p$ for k
3. $7x + 3y = m$ for y
4. $P = 2l + 2w$ for w
5. $A = \frac{1}{2}bh$ for h
6. $A = \frac{1}{2}h(b_1 + b_2)$ for b_1
7. $y = mx + b$ for m
8. $A = \pi r^2$ for r

Word Problems

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a **"let x ="** for each unknown; write an equation; solve the equation; substitute the value for x into the let statements(s) to answer the question.

For Example:

Kara is going to Maui on vacation. She paid $325 for her plane ticket and is spending $125 each night for the hotel. How many nights can she stay in Maui if she has $1200?

Step 1: What are you asked to fine? Let variables represent what you are asked to find.

How many nights can Kara stay in Maui?

Let x = The number of nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$325 + 125x = 1200$

Step 3: Solve the equation for the unknown

$$\begin{aligned} 325 + 125x &= 1200 \\ -325 \qquad\quad &\quad -325 \\ 125x &= 875 \\ x &= 7 \end{aligned}$$

Kara can spend 7 nights in Maui

Word Problem Practice Set

1. A video store charges a one-time membership fee of $12.00 plus $1.50 per video rental. How many videos can Stewart rent if he spends $21?

2. Bicycle city makes custom bicycles. They charge $160 plus $80 for each day that it takes to build the bicycle. If you have $480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?

3. Darel went to the mall and spent $41. He bought several t-shirts that each cost $12 and he bought 1 pair of socks for $5. How many t-shirts did Darel buy?

4. Janet weights 20 pounds more than Anna. If the sum of their weights is 250 pounds, how much does each girl weigh?

5. Three-fourths of the student body attended the pep rally. If there were 1230 students at the pep rally, how many students are there in all?

6. Two-thirds of the Algebra students took the H S A the first time. If 60 students took the algebra H S A how many algebra students are there in all?

7. The current price of a school t-shirt is $10.58. Next year the cost of a t-shirt will be $15.35. How much will the tee shirt increase next year?

8. The school lunch prices are changing next year. The cost of a hot lunch will increase $0.45 from the current price. If the next year's price is $2.60, what did a hot lunch cost this year?

9. Next year the cost of gasoline will increase $1.25 from the current price. If the cost of a gallon of gasoline next year will be $4.50, what is the current price of gasoline?

10. Sarah drove 3 hours more than Michael on their trip to Texas. If the trip took 37 hours, how long did Sarah and Michael each drive?